

What is Einstein's model of a solid?

11 July 2021. A simple model of a solid proposed by Einstein in 1907 is that it consists of a collection of N oscillators with quantized energy units. We can think of each oscillator as a quantum harmonic oscillator, and each energy unit as a quantum of size $h\nu$, but the concept applies to any system with energy units that are all

How many units of energy are distributed in an Einstein solid?

As an example, consider $q=3$ units of energy distributed in an Einstein solid with $N=4$ oscillators. At left is the detailed listing of the possible distributions of the energy, a total of 20 different distributions for 3 units of energy among 4 oscillators (a multiplicity of 20).

How does Einstein model predict heat capacity?

The Einstein model assumes that energy variations in a solid near absolute zero are entirely due to variations in the vibrational energy. From the assumption that all of these vibrational motions are characterized by a single frequency, it predicts the limiting values for the heat capacity of a solid at high and low temperatures.

How do you find the heat capacity of an Einstein solid?

Heat capacity of an Einstein solid as a function of temperature. Experimental value of $3 Nk$ is recovered at high temperatures. The heat capacity of an object at constant volume V is defined through the internal energy U as
$$C_V = \left(\frac{\partial U}{\partial T} \right)_V$$

What is Einstein's theory of heat capacity?

The original theory proposed by Einstein in 1907 has great historical relevance. The heat capacity of solids as predicted by the empirical Dulong-Petit law was required by classical mechanics, the specific heat of solids should be independent of temperature.

How is Einstein temperature determined?

The Einstein temperature is determined by a fit to the measured entropy at 500 K. Experimental data from Barin (1989). Fig. 6.3. As in fig. 6.2 but showing the measured entropy $S(T)$ and heat capacity $C_p(T)$ for TiC. The Einstein temperature is determined by a fit to the measured entropy at 600 K. Experimental data from

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Barin (1989).



Einstein Solid As a last, but important, example of microstates, macrostates and multiplicities, let us consider the Einstein model for a solid. We will use this simple model to make our first attack on the microscopic meaning of temperature and heat. The Einstein model of a solid will, like the ideal gas, be a standard example for many



The example is of an Einstein solid, with $N=3$ oscillators. The book lists the multiplicity of each macrostate, with presumably each macrostate as the total energy units of the system. The Boltzmann constant is a proportionality constant that relates the average energy of a system to its temperature. In the context of an Einstein solid, it



For each of the Einstein solid pairs described in parts (a) through (c), use StatMech to answer the following questions: (1) How many total microstates are available to the combined system? What is the approximate average energy per atom in each solid if the system's macropartition is in one of these most probable bins? (4) What is the

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if the solid is treated as $3N$ independent classical oscillators. Determine the specific heat $C_{1ml V}$ for one mole of substance in this case. (b) When each the solid is treated as $3N$ quantum harmonic oscillators, the energy of the solid is $E = 3N \langle u \rangle$, where the $\langle u \rangle$ is the average energy of the a single harmonic oscillator.



The average energy of the Einstein solid is formulated from the definition of canonical ensemble average and the molar specific heat at constant volume of it is calculated by differentiating the



Key Point 4.24 The Einstein solid's energy and entropy display interesting non-classical behaviour at low temperatures.. 1. The energy $\langle u \rangle$ approaches the zero-point energy, in the zero-temperature limit. That approach follows the expression, $+3N h \nu e^{-h \nu / T}$, and as $T \rightarrow 0$ and $T \rightarrow \infty$, $h \nu / T \rightarrow \infty$, and the exponential is driven to zero. This exponential approach differs from the behaviours of

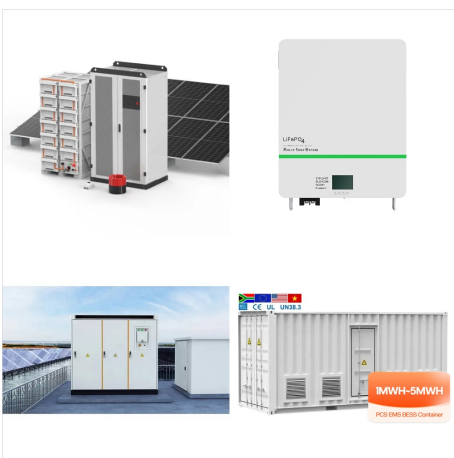
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(a) Use StatMech to generate tables for the following five different Einstein solid pairs. (1) NA-No-50 (2) NA-60, NB-40 (3) NA- 70, N 30 (4) NA-80, NB-20 (5) NA-90, NB-10 In each case, choose $U = 100e$. Compare the average energy per atom (that is, compare U_A/NA to U_B/NB) for the value of U_A corresponding to the peak



Recall that the energy of a single harmonic oscillator in a state with n oscillator quanta is (2) a) Find the partition function of the Einstein solid. b) Find the average energy. c) Find the heat capacity C_v d) Show that at high temperatures, the Einstein model correctly predicts the Dulong-Petit law for the heat capacity of a solid, $C_v = 3Nk_B$



Einstein recognized that Planck's quantization of the molecular oscillators in the walls of the blackbody cavity was, in fact, a universal property of the molecular oscillators in all solids. Accordingly, the average energy of the oscillators was not the $3kT$ of kinetic theory, but rather that derived in Planck's development of the emission

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#energy quanta, since the system is formally equivalent to an Einstein solid (we're distributing the energy quanta among dipoles rather than oscillators). The multiplicity of the paramagnet is then $W \approx \frac{N!}{N_+! N_-!}$ (13) Finally, we can use Stirling's approximation on 2 directly to get an ap-



??? Purpose: This equation specifies the multiplicity Ω of any macrostate of an Einstein solid, where N is the number of atoms in the solid, U is total energy, $q = U/e$ is the number of units of energy to be distribute among the atoms, and $n!$ or n factorial = $1 \cdot 2 \cdot 3 \cdot \dots \cdot n$. ??? Limitations: This equation applies only to an Einstein solid.



(a) Use StatMech to generate tables for the following five different Einstein solid pairs. (1) NA-No-50 (2) NA-60, NB-40 (3) NA- 70, N 30 (4) NA-80, NB-20 (5) NA-90, NB-10 In each case, choose $U = 100e$. Compare the average energy per ???

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In general, a solid with N oscillators can have q energy units to distribute amongst them, so the number of possible microstates of such a system is the number of ways of distributing q balls into N bins. This is a standard problem in combinatorics, and the solution is $\binom{q+N-1}{N-1}$.



The Einstein temperature's accessibility of the vibrational energy inside of a solid molecule determines the heat capacity of that solid. The greater the accessibility the greater the heat capacity. If the vibrational energy is easily accessible the collisions in the molecule have a greater probability of exciting the atom into an upper



Question: (30%) Problem 1: A nanoparticle containing 6 atoms can be modeled approximately as an Einstein solid of independent oscillators. The evenly spaced energy levels of each oscillator are $6e^{-21}$ J apart. Use $k = 1.4e^{-23}$ JK Note that between parts (a) and (b) the average energy increased from "5.5 quanta" to "8.5 quanta".

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Answer to Compute the partition function of a quantum harmonic. Compute the partition function of a quantum harmonic oscillator with frequency ω and energy levels $E_n = \hbar\omega(n + \frac{1}{2})$. Find the average energy E and entropy S as a function of temperature T . Einstein constructed a simple model of a solid as N atoms, each of which vibrates with the same frequency ω .



An Einstein solid is in contact with an external reservoir with temperature T . All oscillators in the Einstein solid are identical with the same energy level spacing $\hbar\omega$. What is its average energy per harmonic oscillator as a function of temperature? Derive this expression from the expression for the entropy that was derived in the class.



The contribution of the three acoustic branches to the average vibrational energy (apart the constant zero point energy) is model for the lattice vibrations (solid curve). The Einstein temperature is determined by a fit to the measured entropy at 500 K. Experimental data from Barin (1989). Fig. 6.3. As in fig. 6.2 but showing the measured

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Einstein solid (20 points) The Einstein solid is a model of a crystalline solid that contains a large number of independent three-dimensional quantum harmonic oscillators of the same frequency ω . We already derived in class and in Problem Set 3 that, for a one-dimensional quantum harmonic oscillator, the average energy is $\langle u \rangle = \frac{1}{2} \hbar \omega + \frac{\hbar \omega}{e^{\beta \hbar \omega} - 1}$ (a)



How can I compute the average energy and the specific heat at constant volume of an Einstein solid? Ultimately, I want to show that the average energy expression obeys the equipartition theorem in the high temperature limit.



TEMPERATURE OF AN EINSTEIN SOLID 2 Since we assumed $U \ll N \epsilon$, this is equivalent to requiring $U = q N \epsilon$, so this result is valid only for low temperatures, as we'd expect. [Schroeder works out the energy-temperature relation for the other extreme $U \gg N \epsilon$ in his section 3.1, with the result $U = NkT$ (10) In this case, there are enough energy

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The mean energy per oscillator is then $u = - \frac{d \ln \Omega}{d \ln \epsilon} = \frac{d \ln \Omega}{d \ln \epsilon} = \frac{1}{2} E + \ln(1 - e^{-\frac{h}{2} E}) = \frac{h}{2} E + \frac{h}{2} e^{-\frac{h}{2} E} - 1$ The first term above, $\frac{h}{2} E$, is simply the zero point energy. Using the fact that energy is an extensive property, the energy of the $3N$ oscillators in ???



Write down the total energy stored in the vibrations of the atoms in an Einstein solid. Explain how quantum mechanical effects influence the energy and heat capacity of solids in the Einstein model. Classical limit of heat capacity. Let us ???